# Investigation of General Equilibrium Modelling for Domestic Sectoral Policy Analysis

#### Disclaimer

The analysis and policy proposals presented in this paper are not government policy and only represents an academic investigation into this policy area.

## Abstract

The UK, like many advanced economies subsides advanced manufacturing sector (Broadberry & Leunig, 2013), particularly if the technology associated are considered ‘Green’. For example, the USA passed the Inflation Reduction Action (IRA), (White House, 2022) and future EU investments will be made by The Green Deal Industrial Plan (GDIP), (European Commission, 2023). There are presumed to be multiply benefits to this decision, however this is a market distortion to a true free market equilibrium. The neo-classical approach (Cordato, 1980) states that without market failures the government should not intervene in the market and that it may produce worse outcomes, government failure (Grand, 2009).The government therefore must show market failures exist that it is trying to counteract and that intervening would be an economic and social benefit to the UK. The first is covered in the Five Case Model, (HM Tresury, 2018). To confirm the latter analysts must perform ex-ante modelling of its impacts (Munby, 2023).

This paper will investigate the potential introduction of General Equilibrium Modelling (GEM) to analysis interventions in advance manufacturing sectors. With a specific focus on the automotive industry due to it being the author’s area of work however there is possibility of applicability to other areas.

## Non-Technical Summary

To assess government policy of intervention into a sector (providing grants or tax relief) economic modelling is completed. The current methodology looks at the potential benefit as the wage premium from increased employment in that sector. It does not model what sectors would have reduced employment and the impacts of on the wider economy.

This paper investigates the use of a Computable General Equilibrium Model (CGEM) to answer the question of impact on other sectors. CGEM attempts to model all of the economy and the interaction between sectors based on the behaviours of economic actors, consumers, firms and the government.

This paper was unable to construct a satisfactory CGEM due to the weakness of the approach. The paper proposes solutions to those weakness and the potential benefits of continuing work on this approach.

## Context

### Government Departmental Changes

The Department for International Trade (DIT) was created on 14th July 2016 and became part of the Department of Business and Trade (DBT) as part of a Machinery of Government (MOG) change on 7th February 2023. DIT had a remit to complete trade policy analysis, for this remit the independent report: Trade modelling review expert panel, recommended development of Computable General Equilibrium Model (CGEM) capabilities, (Venables, 2022).

This remit for external policy focus has been combined with Department for Business, Energy & Industrial Strategy (BEIS) domestic business policy focus. Therefore, allowing government to have a join up domestic and international policy objectives to strengthen UK industry. As DBT matures this joined up policy direction will embed, a requirement for shared analytical approach across the two sides of the departments will grow. Therefore, the domestic policy analysis approach described later in Impact/Benefit justification could be expanded by considering the approach previously used for external policy analysis.

#### Collaboration

After discussing with ex-DIT trade analyst colleagues, the modelling approach investigate in this paper will be based on the work of (Böhringer, et al., 2003) and (Wing, 2004). These discussions formed the original motivation for the work.

### Policy Area - Market Failures

As stated, government needs to conclude that there are market failures before it can intervene. The following section will show there is a strong economic rationale for government intervention in this sector due to the presence of several market failures. Some market failures affect the whole sector, while others are specific to a particular stage, such as R&D investment. Current HMG assessment of different market failures is viewed through the breakdown given in Figure 1. This paper will focus on production side of the industry specifically capital investments. The example of market failures below builds on the information provide in the Green Book 2022, Section 4.23[[1]](#footnote-1).

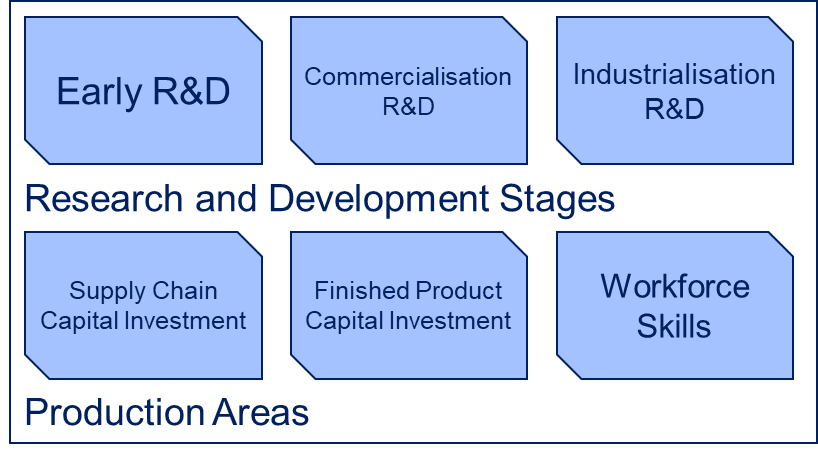


Figure 1 Industry Sections

#### Positive Externality

The industry is current transition to the production of Electric Vehicles (EV) from traditional Internal Combustion Engine (ICEV). EV has a positive externality of reduced CO2 emission when consumed compared to ICEV (USEPA, 2023), even when including the manufacturing process (FCAB, 2021).

As the Marginal Social Benefit (MSB) is greater than the Marginal Private Benefit (MPB) there will be an under consumption and production of the good, (Pindyck & Rubinfeld, 2009). Therefore, the government can intervene to promote the consumption and production of that good.

#### Barriers to Entry and Market Power

For the market to be considered a perfect competition many firms must be present with none having high market power. This is enabled by lower barrier to entry and exit from the market. The automotive industry and its supply chain have high barriers to entry due to the very high initial fixed capital costs; for example, the new gigafactory[[2]](#footnote-2) for JLR will require investment of £4bn (BBC News, 2023).

Certain elements of the electrified supply chain are already characterised by monopolistic supply, for example over 70% of the natural graphite supply is controlled by China.[[3]](#footnote-3)

This market power can enable these Chinese firms to extract profits from European firms by charging above the equilibrium price. The well-established nature of these firms in Asia and the large amount of subsidisation they receive mean that new European firms attempting to entry the market cannot compete.

This uncompetitive international behaviour results in EV supply chain projects typically having low margins[[4]](#footnote-4) and therefore attracting low levels of investable financial especial given the scale required.

#### Information Failures

Although EV have been produced for years, this has not been at the scale required to transfer the entire new car market away from ICEV, a stated goal of the UK government (HMG, 2020) and the EU (Reid, 2023) by 2030 and 2035 respectively. There are multiple competing chemistries that an EV cell can have resulting in uncertainty in investment.

The performance and quality of an EV is driven by this chemistry and the quality of the production process. The knowledge to understand EV cells is complex and therefore the consumers of the good (motorists) will not be able to have accurate information and the market has failed (Pindyck & Rubinfeld, 2009). This can also be true for car manufactures who use the EV cells as intermediate goods in their production process.

Character of the technology (lock-in) – Vehicle technology is very capital intensive, and so exhibits slow replacement rates. This means that the industry will often keep with existing technologies and be too cautious and delay investment until proven alternatives have happened elsewhere before investing significantly into new technologies.

The lock-in nature of the technology means that the following scenario is possible. A cell or car manufacturer has invested in one specific chemistry type which may not develop as productively as other could result in large loss as they are required to retool and invest in the more productive chemistry type. This risk reduces the investment levels in the EV technologies.

### Policy Area – Intervention

The above is a summary of the market failures which were used to justify the Automotive Transformation Fund (ATF), (Munby, 2023), a capital grant fund offering a share of the total budget, valued up to £1 billion, (HMG, 2022) to intervene in the EV supply chain. The total intervention into the sector is called the programme and individual cases are the projects.

#### Impact/Benefit justification

For any grant provided by HMG an ex-ante assessment is completed including an economy appraisal. This is done in line with ‘Green Book’ (GrB) principles. The most common approach is a Cost Benefit Analysis (CBA) appraisal using the Benefit Cost Ratio (BCR) as a metric. The GrB considers the social welfare benefit of bringing advanced manufacturing to the UK as the move to more productive jobs.

This approach assumes that the economy is working at or near full employment and therefore only the wage above the next best alternative wage is a benefit. The next best alternative wage is assessed against a local alternative.

Advance manufacturing like automotive have higher average wages than alternative employment. The majority of advanced manufacturing occurs outside of London and the Southeast aligning interventionist policies with the Levelling Up[[5]](#footnote-5) agenda.

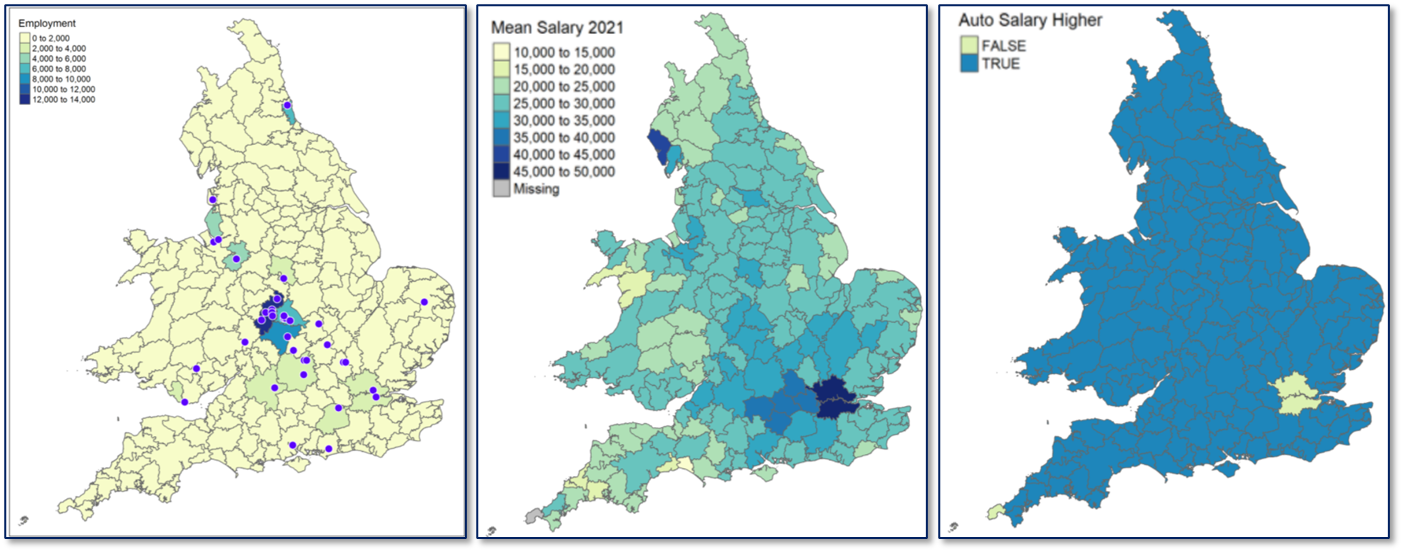


Figure 2 Automotive Employment Maps – ONS data

The maps in Figure 2 show the location of automotive employment, the counterfactual salary, and the areas of the country in which automotive employment represents a move to a more productive employment.

These current assessments are at the project level of individual cases (funding certain OEMs, or key suppliers), but it has been used to approach the programme level intervention. This work will review the macro level of supporting the sector. **This is a live policy debate within HMG and contains commercial sensitive information. Therefore, where appropriate information has been redacted for a wider audience.**

## Methodology

### Current Methodology

The current methodology builds a model of the automotive industry in isolation to the rest of the economy. This methodology is currently in use in the appraisal of an active policy decision; **therefore, where appropriate information has been redacted for a wider audience, include the results of current modelling.** A version of the current methodology was original derived by the author for appraisal of policy of intervention into the automotive sector by HMG and was used to complete the Value for Money assessment presented on gov.uk by the then Permanent Secretary of BEIS, (Munby, 2023).

The model considers the capacity in the UK to produce vehicles, which is a capital-intensive production and is limited by the number of sites of factories. These sites are presented on the first map in Figure 2.

As a result of the change from producing ICEV to EV the new supply chain will be a further limiting factor on the production capabilities in the UK[[6]](#footnote-6). Therefore, investing in both the vehicle manufacturing and EV supply chain new levels of production and employment will be reached. The UK exports the majority (c80% annually) of the vehicles produced and demand for UK produced vehicles is exogenous in this modelling.

It is the modelling described above provides a quantitative assessment of the number of jobs in the sector with and without intervention. The difference in job numbers is multiplied by wage premium associated and is used as the benefit in the CBA. It does not specifically model what the opportunity cost of the economic activity foregone; therefore, it is not a general equilibrium model. There is no consideration given to: what sectors have a reduction in employment, potential changes in output and increases in prices and the wider social-economic impacts.

For this reason, the alternative methodology is proposed and will be investigated for usefulness in this paper.

### Alternative Methodology

The approach being reviewed is a Computable General Equilibrium Model (CGEM), which is a method of constructing a Walrasian general equilibrium in which supply and demand are equalized across all the interconnected markets in the economy, (Wing, 2004).

There is international precedent for the use of CGEM in policy formation, notably Norway and Australia, (Dixon & Jorgenson, 2013). They have been using single-country models since the 1960’s and 1975 respectively. Since the 1990’s other governments and NGOs have built and maintained CGEM to help policy assessment, including US International Trade Commission, the Organisation for Economic Cooperation and Development, and the World Bank, (Burfisher, 2011).

### Uncertainties

A key assumption for both the current methodology and the alternative is that advanced manufacturing sectors will continue to pay a wage premium compared to other sectors. This wage premium is assumed to be a result of the productivity of those sectors. An investigation into this uncertainty can be found in Annex B: Technical: 7 - Wage Premium Uncertainty.

There is uncertainty that the data used in the CGEM will be detailed enough to account for the production change from ICEV to EV. There is a further discussion of this point in Generality of Production Function.

### Project Aims

To investigate the possibility and usefulness of developing a CGEM to analysis programme level intervention into advance manufacturing sector. The worked example will be the automotive sector and its transition from ICEV to EV production.

### Output Objectives

The output aims for a complete model would be:

* Investigate the structures presented in (Wing, 2004),
* Construct a method to establish the distribution of employment between different sectors,
  + With and without policy intervention
* The comparative salary distribution,
* Understand the distribution of wages across areas,
* Produce a framework to analysis regional equality impacts.

Motivation - Objectives

The reasons for these aims are:

* To enable the model to produce a counterfactual to compare to the intervention state to calculate the impact of the policy.
* To enable comparison of modelling results on the same primary economy benefit (wage premium).
* To assess policy impacts against wider strategy of Levelling Up.

The benefit of taking an CGEM approach would be to investigate the opportunity cost of the economic activity foregone. It would consider how the government intervention will impact the different sectors of the economy.

### Economy structure - closed circular economy

A diagram of a company

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Figure 3 Close circular economy model

The underpinning idea in CGEM is circular flow of commodities in a closed economy. The above model diagram is an autarky version of the model present in the HMRC GEM paper, (HMRC, 2013) .

CGEM can be dynamic and track the evolution of the economy over time in response to a policy change. In the baseline, it is assumed the economy follows a steady-state growth path where all economic activities grow at a constant rate, (HMRC, 2013).

### Required Modelling Assumptions

The following are require modelling assumptions to build a working CGEM.

#### Firms Assumptions

Firms are assumed:

1. To be profit maximises,
2. To be producing homogenous products,
3. To have a homogenous production function within a single sector of the economy,
4. To be producing using labour, capital, and intermittent good as input factors,
5. To be producing with a constant return to scale,
6. To have production modellable with a Cobbs-Douglas (CD) function.

#### Households Assumptions

Households are assumed:

1. To be utility maximisers, with a define Utility Function (UF),
2. To gain income from selling labour as price takers, and collecting interest on invested capital at fixed rate,
3. To save income at a fixed rate,
4. To be budget neutral in a single period; income = expenditure + savings.

#### Government Assumptions

The government is assumed:

1. To have a constant tax and spend policy throughout model expect for the policy being assessed.

### Justification and Reasoning

The reason for some of the assumptions above is such that the CGEM is a modelling a perfectly competitive market; specifically, 1, 2, 3, 7, & 8. They are as justifiable as assumption in a model as assuming that the UK economy is a perfectly competitive market is justifiable. The scope of this paper does not include discuss this whether the UK is a perfectly competitive market. The reasoning and justification for the other assumptions are given in Table 1 below.

|  |  |  |
| --- | --- | --- |
| **Assumption** | **Reasoning** | **Justification** |
| 4 | The inclusion of these three factors allows for a model which explores the topics of interest for the policy area. Labour is required to understand wage premium impacts, capital to understand how government intervention could be achieved and the intermediate goods are included to understand interaction between the different sector of the economy. | The production function is limited to 3 factors and excludes entrepreneurship and land as they would increase complexity without provide insight for the policy question. |
| 5 | This helps with analytical simplification, specifically in the regression modelling of the CD production function. | This is a widely used assumption in analysis of production (Lim & Shumway, 1992) however if not valid can lead to mistakes in output combination, one example is between risk-neutral optimising firms and those which are risk averse. However, these considerations are outside the scope of this paper therefore the paper will assume the simplifying assumption. |
| 6 | Ensure that the production function is algebraically manageable for the modelling and while enabling it to contain the necessary factors of production. | There would be limited benefits to the scope of the policy question for incorporate increase complexity. |
| 9 | Analytical simplification to make this saving rate exogenous to the model | There would be limited benefits to the scope of the policy question for incorporate increase complexity |
| 10 | Analytical simplification to reduce the modelling requirements. | There would be limited benefits to the scope of the policy question for incorporate increase complexity |
| 11 | Analytical simplification and would allow the model to be computable | There would be limited benefits to the scope of the policy question for incorporate increase complexity |

Table 1 Modelling Assumptions - Reasoning and Justifications

## Modelling – CGEM

The data on which the CGEM were calculated with is from the ONS – Office for National Statistics. ONS produces Input-Output Analytical Tables which includes product-by-industry section derived from the annual Supply and Use Tables (SUTs) (ONS, 2023). From this the annual use of intermediate goods, labour expenditure and final output can be gathered. This data has disaggregated by SIC code as described in Annex A: Standard Industrial Classification (SIC). This classification of sectors has used at two levels, 39 sectors and 5 sectors.

This data is the appropriate set for this work as it gives the depth of disaggregation (covers a range of sectors), breadth of data points (range of years) and is relative consistent in presentation when matching across datasets.

### Model Derivation

Initial modelling derivation was built from the work presented in the MIT Joint Program on Science and Policy of Global Change report: Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis. (Wing, 2004).

Set of commodities:

Set of industry sectors:

Set of primary factors:

Set of final demands:

For simplicity

In CGEM, the price and income elasticities of demand are crucial in determining the sectoral growth pattern and economic impact, (Hertel, 2012). For this initial model the UF, is modelled as a CD, from this function the price elasticities of demand can be calculated, this is discussed further in the UF section.

It assumes that the circular economy in the model conforms to:

* Cobb-Douglas Economy
* Commodity market clearance
* Factor market clearance
* Full employment
* Zero profit

These assumptions can be formulised as in the equations below:

|  |  |
| --- | --- |
|  | Equation 1 Commodity Market Clearance |
|  | Equation 2 Factor Market Clearance |
|  | Equation 3 Full Employment |
|  | Equation 4 Zero Profit |

Where:

|  |  |
| --- | --- |
| **Variable** | **Definition** |
|  | Total output of commodity |
|  | Intermediate good required for commodity |
|  | Final good use of commodity from factor rent |
|  | Consumption of commodity |
|  | Saving of commodity |
|  | Total available primary factor |
|  | Primary factor used for sector |
|  | Total rent from all primary factors employed |

The households in this model are assumed to be utility maximises (Assumption 8) and therefore the agent problem can be defined as:

|  |  |
| --- | --- |
|  | Equation 5 Agent Utility Maximisation |

The where the function is defined as:

|  |  |
| --- | --- |
|  | Equation 6 Cobb-Douglas Utility Function |

This can be solved to results in the equations for

|  |  |
| --- | --- |
|  | Equation 7 Consumption of commodity i |
|  | Equation 8 Alpha of commodity i |

The producers in this are model are assumed to be profit maximises (Assumption 1) and therefore the producer’s problem can be defined as:

Equation 9 Produces Profit Maximisation

Where is specified as the production function. For this model it will be an extension of the CD investigated in Equation 22 to:

Equation 10 Produces Cobb-Douglas with Intermediate Goods

|  |  |
| --- | --- |
| Where: | Equation 11 Constant Return to Scale |

From these equations can be derived as:

The aim is to create a benchmark within which price of each commodity and wage are set to 1. This unity of prices and wages is a modelling assumption to enable algebraic simplification without the loss of generality in the modelling. There is no loss in generality of modelling as the prices and wages are relative to the base year and consumption/output is not modelled as unity. The following are the equations used to calculate the calibration parameters:

|  |  |
| --- | --- |
|  |  |
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|  |  |

Equation 12 Calibration Equations

With the above definitions for calibration the general equilibrium equations can be reformulated as:

|  |  |
| --- | --- |
|  | Equation 13 Benchmark Commodity Market Clearance |
|  | Equation 14 Benchmark Factor Market Clearance |
|  | Equation 15 Benchmark Full Employment |
|  | Equation 16 Benchmark Zero Profit |

Rearranging Equation 13 to Equation 16 and inserting the calibration definitions from Equation 12, the following deltas can be derived:

|  |  |
| --- | --- |
|  | Equation 17 Delta Consumption |
|  | Equation 18 Delta Primary Factor |
|  | Equation 19 Delta Profit |
|  | Equation 20 Delta Factor Clearance |

This delta function can be combined into a single function of function:

|  |  |
| --- | --- |
|  | Equation 21 Delta Function |

For a given set of calibration inputs a vector is in equilibrium

The difference of from can be interrupted as the amount the vector needs to change to move the model to equilibrium.

#### Structural Choices

There is a trade-off between the computable & producibility of the model and the complexity & number of features in the model. These options must be balanced when constructing the model. Examples of this include the number of sectors in the model, the functions deriving economic actor behaviour.

### Model 1

For the model to more computable with initial modelling the 5-sector disaggregation has been used. For this 2015 ONS Input-Output Table[[7]](#footnote-7) data was taken and a benchmark CGEM was constructed. This version of the model assumed no savings, capital, imports, exports, or government involvement.

However, this is not the case in the data from ONS which are presented in current price year opposed at a unit production level. So, the outputs had to themselves calibrated to unity price and wages, which is presented in the code: 11\_Model\_1\_Calibration.R.

The ONS data provide a unit input-output for intermediate goods which could be used for the within the model. From this a corresponding set of and . The model constructed its own (technology constant) to allow each sector to create enough output to cover both intermediate goods and final demand .

Implementing this approach, the benchmark model was able to reach general equilibrium as within the realms of computational tolerance, account for floating point errors.

This could be tested with the delta function in Equation 21.

### Model 2

For a more complex version of the model capital will be introduced as an additional required primary factor. The construction of the first began again with the ONS data, with an initial endowment of fixed capital levels in 2015 and employment figures.

The total output was scale to match a salary cost of 1 and capital cost of 0.1 (to represent an economic wide interest rate of 0.1). This model produced a set of outputs include intermediate goods required and final consumption levels for consumers which match the return on labour and capital, . However, as the initial input data is based on real world values, with the additionality complexity of two primary factors the model could not be engineered such that all the deltas were zero.

### Model 2 Next Steps

The next test for the model is varying the initial exogenous variables and so the benchmark model comes to an equilibrium. This requires the creation of an algorithm to search for the equilibrium point. The endogenous variables in Model 2 that could be altered are the vectors of price , wages , and consumption (consumption alters total output ), each of which are containing 5 elements.

This gives 15 variables which could be scaled, and table below present the relationship between the deltas and increase in the variables. When the delta is has a subscript this means the change of a delta for a different commodity.

|  |  |  |  |
| --- | --- | --- | --- |
| **Increase** in Variable |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

The literature recommends an approach similar to a Newton-type steepest-descent optimization algorithm (Kehoe, 1991). An attempt to construct an algorithm to find the equilibrium can be found in the R code: Model\_2\_Calibration.R. a flow diagram of the attempt is given in Figure 4. The function Update Values contain the Newton-type steepest-descent. However, this was unsuccessful.

A diagram of values and functions

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Figure 4 Flow Diagram for Equilibrium search

## Modelling Weaknesses

Although the CGEM can investigate the opportunity costs of the intervention better than the previous approaches by showing which industries are impacted by incentivising the Automotive sector it does have some weaknesses. This section of the paper will discuss them and give some potential solutions where investigated.

### Weaknesses with the Benchmark Approach

The following explores the weaknesses with the Benchmark aspect of the CGEM approach.

#### Unity Prices and Wages

The methodology reviewed in the documentation readily used in DBT (ex-DIT) assumes in the benchmark year prices and wages are set to 1. This enables the algebra to construct future years relative to the first year. However, this assumption negates the main benefit that a model would be trying to predict. Therefore, an attempt to construct a version that assumes wages which are not equal to 1 was created. The code can be found in 00\_Main\_Model\_v1.R[[8]](#footnote-8). The code was unable to find the equilibrium point for this version. This version also attempted to model all 39 SIC commodity groups as described in Annex A: Technical: 1 Standard Industrial Classification (SIC).

#### Production Function

The methodology presented by (Wing, 2004) and used in the modelling above calculates the CD coefficients for production from a single year of data. Depending on the sensitivity of the results to these coefficients this may lend to misleading results. A potential solution to this is approximate the CD from many years of data.

The three factor Cobb-Douglas is given as:

Where: Output, intermediate goods, capital used, labour used, technology constant.

Equation 22 Factor CD Production Function

The would as be inferred from the ONS Input-Output tables. Linear regression modelling and an author derived data searching algorithms were used to calculate the parameters of Equation 22. Full technical detail of the approach can be found in Annex A: Technical.

The benefit of this approach is to produce more robust coefficients.

### Weaknesses with the General CGEM Approach

The following explores the weaknesses with the CGEM approach, additional to those in caused by benchmarking.

#### Data Limitations – SIC Codes

The SIC code 29 covers the production in the automotive industry, a breakdown is given in Figure 5. There is currently no split between the production of ICEV and EV which are both contained within SIC 29. A screenshot of a computer

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Figure 5 SIC 29 Breakdown

This means that any production function derived from SIC 29 will have capitalisation levels and intermediate goods for both ICEV and EV. The UK currently produces a small percentage of EV compared to ICEV which means that it can be assumed that production function is likely representative of the current sector and for the next few years. However, as the industry transitions to produce more EV this assumption will be weakened.

Below is a proposed method to handle this is when calculating the production function by the regression given in Annex A: Technical 3. Production Function 1.

The regression model will have to incorporate a term for the percentage of the market which is EV in that year. This will try to determine how the intermediate goods and capitalisation level change as EV proportion increases.

This could allow for a model with an exogenous term of EV percentage which alerts the production function over time.

Data usefulness

#### Generality of Production Function

The parameters of Equation 22 provide a more robust production function than the one used in Model 1 and 2. However this the approach still suffers from the data limitation described above; namely the CGEM approach is based on data which includes both the production of ICEV and EV as the data is not disaggregated between the two.

The current methodology is able to model the balance between investing in the EV supply chain and at the car manufacturing level. For this to be incorporated into the CGEM the commodities used would have to be disaggregated further to specifically model the EV supply chain. This is not possible with current data source (ONS, Input-Output Tables). The current methodology relies on specific subject matter expertise which could not be duplicated for the CGEM approach.

Therefore, the alternative methodology explored in this paper is not as accurate as the current method.

#### Multiplicity of Equilibrium

It is extremely difficult to proof analytically that a given equilibrium solution is unique in real world models, especial when there is the inclusion of taxes distortion. There have been examples testing for multiplicity of equilibria in real-world CGE models. However, the complexity of doing so would means there are uncommon. (Wing, 2004).

This does provide a doubt on the solution and subsequential policy advice provided by any CGE modelling.

#### Utility Function (UF)

There are multiple implications for modelled consumer behaviour by using the CD UF which are discuss in this section by investigating different elasticities. These include Price elasticity of demand, Cross price elasticity of demand and Income elasticity of demand.

##### Price elasticity of demand

The price elasticity of demand is defined as , (Mac Lean, 2011) can be calculated by differentiating Equation 7: with respect to . In model 1 and 2 described above there was assumed to be no savings which means for those models it gives:

|  |  |
| --- | --- |
|  | Equation 23 Price elasticity for Model with no savings |

The derivate as , this implies that the demand will always decrease for increasing prices and vice versa. This means that Giffen goods cannot be included in the model.

The price elasticity of a model including savings is evaluated below, assuming is not dependent on .

Equation 24 Price elasticity for Model with savings

The modelling assumes that the consumer budget is balanced within a single year (no borrowing), therefore Therefore the price elasticity in a model has the same behaviour, i.e., no Giffen goods.

##### Cross-price elasticity of demand

Cross-price elasticity of demand is defined as (Pindyck & Rubinfeld, 2009)

In the model with no saving there is no cross elasticity of demand as .

The cross elasticity of a model including savings is evaluated below, assuming is not dependent on .

|  |  |
| --- | --- |
|  | Equation 25 Cross Elasticity in Model with Savings |

As price are defined as positive at any solution to the model, Equation 25 implies that all cross elasticities are negative and therefore all goods are complementary.

##### Income elasticity of demand

Income elasticity is defined as , where is income, (Pindyck & Rubinfeld, 2009). Income elasticity is the same in a model with constant saving rate (Assumption 9, ) or without savings:

|  |  |
| --- | --- |
|  | Equation 26 Income Elasticity |

This implies that an increase in income will always results in a linear increase in consumption of all goods. As an example of this model behaviour, Figure 6 has been created for a three good economy with constant return to scales in CD with: . This assumes all other factors remain equal (e.g., price of commodities.)

Figure 6 CD predicted spend as income changes.

This implication can be tested against real world data via the ONS 'Household expenditure by gross income decile group’ dataset[[9]](#footnote-9). Comparing the total spend on each commodity group in each decile to the lower decile one (i.e., Third decile group compared to Second decile group) you can infer the income elasticity. The assumption is that if individuals earning the salary in second decile moved to the income of third decile, they would shift that spend on average to that of the third decile.

Reviewing the data from ONS infers that there is evidence to support a negative income elasticity at certain income levels for:

Alcoholic drinks, tobacco & narcotics,  
Housing(net)[[10]](#footnote-10), fuel & power,  
Health &  
Education[[11]](#footnote-11)

This is evidence by Table 2 which shows the percentage of years that a negative elasticity is calculated when moving decile group for each commodity group. This is measuring absolute spend decreases not just a proportion decrease.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Moving from Percentile X to Y | | | | | | | | |
| Commodity Group | 10 to 20 | 20 to 30 | 30 to 40 | 40 to 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 |
| 1 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| 2 | 33% | 17% | 8% | 0% | 0% | 17% | 8% | 42% | 0% |
| 3 | 0% | 0% | 0% | 0% | 8% | 8% | 0% | 0% | 0% |
| 4 | 0% | 8% | 0% | 25% | 17% | 50% | 50% | 50% | 0% |
| 5 | 8% | 0% | 17% | 17% | 0% | 25% | 25% | 8% | 0% |
| 6 | 0% | 42% | 8% | 33% | 33% | 42% | 33% | 8% | 8% |
| 7 | 0% | 0% | 0% | 0% | 0% | 8% | 0% | 0% | 0% |
| 8 | 0% | 0% | 0% | 0% | 8% | 0% | 0% | 0% | 0% |
| 9 | 0% | 0% | 0% | 0% | 0% | 8% | 0% | 0% | 0% |
| 10 | 67% | 42% | 17% | 42% | 75% | 0% | 42% | 42% | 0% |
| 11 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| 12 | 0% | 0% | 0% | 8% | 0% | 17% | 0% | 0% | 0% |

Table 2 Result of investigating commodity groups with negative income elasticities.

To examine the income elasticity with this data requires the assumption that firms within a commodity group are producing homogenous goods/services. This means that the prices paid for a unit of goods/service does not change. This removes the possibility that consumer will pay for a more luxury version of the same item as income increases, i.e., changing from Supermarket own brand to a premium brand food product. The inverse of this phenomenon has been seen in the post-COVID cost-of-living crisis, with 64% of consumers are switching to cheaper brands (Grocery trader, 2022) according to a Shopmium survey[[12]](#footnote-12).

To explore behaviour predicted by the CD UF will be tested against data further with regression modelling of UK consumption data, this is presented in Regression Modelling of Consumption.

### Autarky

The new methodology models the economy as an autarky excluding international trade. This was by design to enable an initial exploration of the model limiting complexity. For any complete usable version of model for real policy analysis would have to consider international trade. In the last ten years, despite the reduction in vehicle production numbers from a high in 2016, the percentage of exports has remained around 80%. This is evidenced in data presented in Figure 4[[13]](#footnote-13)

Figure 7 UK Vehicle Production Data

This is an area that a future CGEM could be stronger than the current approach. The current approach assumes that international demand for vehicles is exogenous and will absorb any increase in UK production. A CGEM will be able to test this assumption and build in trade reactions.

An international trade CGEM will also be able to contextualise UK government policy intervention with USA IRA and EU GDIP policies. However, this development option will require a considerable larger dataset and calibration.

### Black Box

Simulation models like CGEM are criticized for being ‘Black Boxes’ (Burfisher, 2011), whose results are difficult to explain by clear causal chains (Dixon & Jorgenson, 2013). This is especial true for proprietary commercial modelling, which is the reason is the reason that R an open-source coding language was attempted when creating a new CGEM.

One of the key approaches to overcome this weakness is in the presentation approach. The recommendation would be:

* Narrowing the scope of the results to key areas of interest to the audience
* Focus on understand how policy intervention impact key metrics (employment, trade ratios).

## Regression Modelling of Consumption

It should be noted the following estimators from regression models are not the elasticities the sign (- or +) does indicate the sign of the elasticities. To understand with the commodities are inelastic or elastic the current price level and quantity consumed would have to be incorporated into the equations.

### Income Elasticity of Demand

The ONS 'Household expenditure by gross income decile group’ dataset provides multiple years of data which can be compared together when accounting for the different price years. The income in each year becomes comparable once the GDP deflator[[14]](#footnote-14) has been used to bring all data points to a single price year. The relative quantity change was calculated by assuming a unit price of 1 in 2008. Quantity consumed of commodity was given by:

|  |  |
| --- | --- |
|  | Equation 27 Relative Quantity Consumed |

was taken from ONS data[[15]](#footnote-15), this data covers 2008-2019.

A collage of graphs

Description automatically generated

Figure 8 Plot of monthly quantity consumed of commodities by income level from 2008-2019

Regression analysis can be run on this data to test the implication of the CD of linear relationship between total income and commodity quantity. As it is an area of specific interest in this project Group 7 – Transport has been split into 7.1 Automotive purchases and 7-other (all other consumption in Group 7), results shown in Figure 8.

A comparison of a graph

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Figure 9 Plot of monthly quantity consumed by income level for disaggregated Transport group.

Simple Linear Regression

Equation 28 1st Income Regression - Simple Linear Regression

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Commodity | Intercept |  |  |  |  | Adjusted |
| Commodity 1 | 17.8 | 4.18E-50 | 0.066 | 5.37E-76 | 0.878 | 0.877 |
| Commodity 2 | 3.80 | 2.36E-39 | 0.011 | 4.16E-57 | 0.566 | 0.562 |
| Commodity 3 | -3.98 | 1.11E-12 | 0.064 | 5.94E-88 | 0.954 | 0.953 |
| Commodity 4 | 33.0 | 8.55E-58 | 0.049 | 5.51E-43 | 0.789 | 0.787 |
| Commodity 5 | -2.88 | 7.67E-05 | 0.074 | 5.42E-74 | 0.932 | 0.932 |
| Commodity 6 | 0.179 | 0.587 | 0.012 | 1.46E-47 | 0.818 | 0.817 |
| Commodity 7.1 | -9.52 | 6.33E-20 | 0.072 | 6.62E-62 | 0.938 | 0.938 |
| Commodity 7 - Other | -8.53 | 5.72E-29 | 0.110 | 3.12E-96 | 0.968 | 0.967 |
| Commodity 8 | 3.89 | 3.93E-22 | 0.019 | 6.02E-55 | 0.924 | 0.923 |
| Commodity 9 | -11.2 | 9.90E-30 | 0.169 | 1.20E-116 | 0.98 | 0.98 |
| Commodity 10 | -5.97 | 3.18E-09 | 0.028 | 1.35E-25 | 0.628 | 0.625 |
| Commodity 11 | -10.5 | 1.79E-35 | 0.109 | 3.73E-105 | 0.944 | 0.943 |
| Commodity 12 | -4.06 | 1.22E-13 | 0.091 | 1.97E-105 | 0.964 | 0.963 |

Table 3 1st Regression Results

By simply modelling the relationship between quantity and income as a linear relationship yields good results for certain commodities. This inferred by the very low on all the Intercepts and expect commodity 6’s Intercept. The and adjusted are also very high for all commodities (except 10 and 4).

This model could be improved by including the assumption that the intercept is zero. This is a reasonable assumption as it implies without an income that an individual does not consume any commodities.

Linear Regression with zero intercept

Equation 29 2nd Income Regression – Linear regression with zero intercept

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Commodity |  |  |  | Adjusted |
| Commodity 1 | 0.102 | 4.5 E-92 | 0.962 | 0.962 |
| Commodity 2 | 0.019 | 4.8 E-85 | 0.919 | 0.918 |
| Commodity 3 | 0.056 | 2 E-113 | 0.984 | 0.984 |
| Commodity 4 | 0.115 | 8.7 E-69 | 0.921 | 0.921 |
| Commodity 5 | 0.068 | 1 E-108 | 0.983 | 0.983 |
| Commodity 6 | 0.012 | 1.9 E-88 | 0.962 | 0.962 |
| Commodity 7.1 | 0.053 | 5.8 E-67 | 0.951 | 0.951 |
| Commodity 7 - Other | 0.093 | 1.6 E-98 | 0.984 | 0.984 |
| Commodity 8 | 0.027 | 5 E-92 | 0.974 | 0.973 |
| Commodity 9 | 0.147 | 4 E-125 | 0.990 | 0.990 |
| Commodity 10 | 0.017 | 1.8 E-30 | 0.699 | 0.696 |
| Commodity 11 | 0.088 | 7 E-104 | 0.971 | 0.970 |
| Commodity 12 | 0.083 | 3 E-132 | 0.989 | 0.989 |

Table 4 2nd Regression Results

All of the and adjusted values have improved, showing a potentially better model. The statistical confidence of the values is very high as the all the are very low (significant less than 0.001). This implies this model is a good fit. To confirm that a linear regression with a zero intercept is a better an Analysis of Variance (ANOVA) test was completed with the simple regression set as the null hypothesis. If the resulting is less than 0.001 this is considered a significant result to reject the Null hypothesis, i.e., the Linear regression with zero intercept is a statistically significant better model.

|  |  |
| --- | --- |
| Commodity |  |
| Commodity 1 | 2.27 E-34 |
| Commodity 2 | 1.35 E-15 |
| Commodity 3 | 5.48 E-10 |
| Commodity 4 | 1.64 E-59 |
| Commodity 5 | 0.00067 |
| Commodity 6 | 0.450 |
| Commodity 7.1 | 2.54 E-19 |
| Commodity 7 - Other | 4.70 E-15 |
| Commodity 8 | 1.70 E-33 |
| Commodity 9 | 3.01 E-20 |
| Commodity 10 | 2.92 E-09 |
| Commodity 11 | 2.92 E-16 |
| Commodity 12 | 2.02 E-07 |

Table 5 ANOVA Results – 1st vs 2nd Regressions

Therefore, for all (expect for Commodity 6) the ANOVA test implies that there is significant statistically evidence to reject Null hypothesis and use the second regression.

There is still limited evidence to support rejecting null hypothesis, but this could be less statistically significant because the Simple Regression modelling producing an intercept close to zero (0.179). Therefore, hard to test the small change in the intercept between regression models. When considering it is still a positive result from the ANOVA and there is improvement to of , and adjusted the Null hypothesis should be rejected.

This regression evidence so far supports the modelled behaviour of the CD UF. It should be noted that the sum of in the Linear Regression model with zero intercept is 0.88 which is potential can be expand by an increase marginal rate of savings as income increases.

Visual inspection of the charts in Figure 8 infers that a logarithmic regression should be tried. This would imply that for some of the commodities as income increases the proportion of income spent on them decreases. Implying the marginal utility of consumption of those commodities decreases relative to the marginal consumption of other commodities as income increases if those other commodities are shown to still have linear relationships.

Log Regression

Equation 30 3rd Income Regression – Logarithmic

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Commodity | Intercept |  |  |  |  | Adjusted |
| Commodity 1 | -112.6 | 2 E-76 | 26.7 | 1.1 E-92 | 0.905 | 0.904 |
| Commodity 2 | -18.6 | 1.7 E-45 | 4.58 | 2.9 E-61 | 0.584 | 0.580 |
| Commodity 3 | -123 | 6.9 E-52 | 24.7 | 1.8 E-59 | 0.884 | 0.883 |
| Commodity 4 | -64.8 | 4.3 E-28 | 20 | 4.6 E-50 | 0.84 | 0.839 |
| Commodity 5 | -136.5 | 1.4 E-40 | 27.8 | 3.5 E-48 | 0.83 | 0.829 |
| Commodity 6 | -22.8 | 7.1 E-36 | 4.74 | 6.7 E-44 | 0.793 | 0.791 |
| Commodity 7.1 | -143.7 | 6.4 E-40 | 27.7 | 9.3 E-45 | 0.866 | 0.864 |
| Commodity 7 - Other | -215.7 | 9.8 E-52 | 42.7 | 6.3 E-58 | 0.911 | 0.910 |
| Commodity 8 | -34.4 | 2.5 E-50 | 7.82 | 2 E-63 | 0.965 | 0.965 |
| Commodity 9 | -322 | 6.2 E-55 | 64.4 | 1.5 E-62 | 0.900 | 0.899 |
| Commodity 10 | -52.3 | 1.2 E-14 | 9.83 | 6.6 E-17 | 0.463 | 0.459 |
| Commodity 11 | -208.4 | 6.3 E-51 | 41.1 | 1.1 E-57 | 0.855 | 0.854 |
| Commodity 12 | -172.3 | 1.6 E-54 | 34.8 | 1.3 E-62 | 0.890 | 0.889 |

Table 6 3rd Regression Results

It is not expected that this model will perform better than the 2nd regression for all commodities. The ANOVA test was used again to confirm which commodities are likely to have Logarithmic relationships. If an ANOVA test results in an NA there is no evidence to reject the Null Hypothesis (Linear Regression with Zero intercept).

|  |  |
| --- | --- |
| Commodity |  |
| Commodity 1 | 8E-67 |
| Commodity 2 | 1.6E-43 |
| Commodity 3 | NA |
| Commodity 4 | 7.2E-65 |
| Commodity 5 | NA |
| Commodity 6 | NA |
| Commodity 7.1 | 0.0255 |
| Commodity 7 - Other | NA |
| Commodity 8 | 1.2E-30 |
| Commodity 9 | NA |
| Commodity 10 | NA |
| Commodity 11 | NA |
| Commodity 12 | NA |

Table 7 ANOVA Results – 2nd vs 3rd Regression

For commodity 1,2,4 and 8 there is significant statistical evidence to reject the Null hypothesis. For Commodity 7.1 the evidence is no significant enough to reject and the Linear Regression with Zero Intercept has better , and adjusted results. Therefore, for all other commodities (3, 5, 6, 7.1, 7 – Other, 9, 10, 11 and 12) the Null Hypothesis is not rejected. The final relationships are given in Table 8 and example charts of the regression and data are given in Figure 9, with all charts in

|  |  |  |
| --- | --- | --- |
| Code | Group (Description) | Relationship |
| 1 | Food & non-alcoholic drinks | Logarithmic |
| 2 | Alcoholic drinks, tobacco & narcotics | Logarithmic |
| 3 | Clothing & footwear | Linear |
| 4 | Housing(net)[[16]](#footnote-16), fuel & power | Logarithmic |
| 5 | Household goods & services | Linear |
| 6 | Health | Linear |
| 7.1 | Transport – Automotive | Linear |
| 7 - Other | Transport – Other | Linear |
| 8 | Communication | Logarithmic |
| 9 | Recreation & culture | Linear |
| 10 | Education | Linear |
| 11 | Restaurants & hotels | Linear |
| 12 | Miscellaneous goods & services | Linear |

Table 8 Income Regression Modelling Results

Logarithmic relationship between income and quantity implies a non-constant elasticity which are not possible with the CD UF which are a weakness when comparing to the available data. Specifically, as the modelling objective is investigating the changes to the economy of subsiding and incentivising advance manufacturing as it has a wage premium and therefore impacts on consumption with higher incomes.

A graph with blue dots and red dots

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Figure 10 Example of Income Regression Results and Data

A guide to find all charts produced for paper can be found in Annex B: Code Blocks. The income model for quantity of commodities consumed can be shown to be a good fit and statistically significant it will not explain all the quantity changes, expressed in the regression as the error term .

### Price Elasticity of Demand

Following the same approach as income elasticity of demand a simple regression model of price vs spend was constructed.

Equation 31 1st Price Regression - Linear.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Commodity | Intercept |  |  |  |  | Adjusted |
| Commodity 1 | 85.3 | 0.05827 | -0.419 | 0.73034 | 0.035 | 0.026 |
| Commodity 2 | 18.3 | 4.2E-09 | -0.106 | 0.4576 | 0.267 | 0.261 |
| Commodity 3 | 48.6 | 0.54299 | -0.265 | 0.92517 | 0.003 | -0.005 |
| Commodity 4 | 70.3 | 0.10724 | -0.185 | 0.00171 | 0.022 | 0.014 |
| Commodity 5 | 29.9 | 0.95335 | -0.028 | 0.27006 | 0 | -0.008 |
| Commodity 6 | 5.81 | 0.76177 | -0.008 | 0.11753 | 0.001 | -0.008 |
| Commodity 7.1 | 32.1 | 0.90645 | -0.12 | 0.91639 | 0 | -0.01 |
| Commodity 7 - Other | 62.6 | 0.53484 | -0.256 | 0.79101 | 0.004 | -0.006 |
| Commodity 8 | 9.28 | 0.50694 | 0.025 | 0.00058 | 0.003 | -0.005 |
| Commodity 9 | 56.9 | 0.9942 | 0.005 | 0.53417 | 0 | -0.008 |
| Commodity 10 | 7.02 | 0.83964 | -0.015 | 0.05318 | 0.002 | -0.007 |
| Commodity 11 | 51.8 | 0.41793 | -0.188 | 0.43696 | 0.006 | -0.002 |
| Commodity 12 | 53.6 | 0.61466 | -0.21 | 0.73758 | 0.002 | -0.006 |

Table 9 1st Price Regression Results

The price regression model did not yield statistically significant results and the chart of price vs quantity data indicates why. There are significant fixed effects in the model, namely the income effect on quantity. The previous regression model has shown the statistical significance of this effect, therefore a Fixed Effect model had to be implemented.

A screenshot of a graph

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Figure 11 Price vs Spend Data

#### Fixed Effects Regression

Equation 32 Price vs Quantity fixed effect regression.

The income level is fixed within the model by fixing the income decile group across years.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Commodity |  |  |  |  | Adjusted |
| Commodity 1 | -0.419 | 1.9 E-30 | 0.985 |  | 0.985 |
| Commodity 2 | -0.106 | 1.9 E-41 | 0.938 |  | 0.938 |
| Commodity 3 | -0.265 | 7.9 E-05 | 0.979 |  | 0.979 |
| Commodity 4 | -0.185 | 6.8 E-08 | 0.929 |  | 0.929 |
| Commodity 5 | -0.028 | 0.649 | 0.939 |  | 0.939 |
| Commodity 6 | -0.008 | 0.526 | 0.838 |  | 0.838 |
| Commodity 7.1 | -0.120 | 0.634 | 0.926 |  | 0.926 |
| Commodity 7 - Other | -0.256 | 4.4 E-09 | 0.991 |  | 0.991 |
| Commodity 8 | 0.025 | 0.002 | 0.967 |  | 0.967 |
| Commodity 9 | 0.005 | 0.961 | 0.988 |  | 0.988 |
| Commodity 10 | -0.021 | 0.203 | 0.736 |  | 0.736 |
| Commodity 11 | -0.188 | 6.2 E-08 | 0.980 |  | 0.980 |
| Commodity 12 | -0.210 | 5.8 E-05 | 0.985 |  | 0.985 |

Table 10 2nd Price Regression Results

This model produces more statistically significant results. It also has shown an expected result, that all the in the model are negative. This implies that commodities groups on average have the property as price increases the quantity consumed decrease. Although not all the are statistically significant which could be expanded by too few data points or that the consumption behaviour of the commodity is more dependent on the Cross-Price Elasticity. These commodities include 5, 6, 7.1, 9 and 10.

### LASSO and Ridge Regression

To investigate the Cross Price Elasticity a Multi Regression method could be used one of the assumptions of this model type is ‘No perfect Multicollinearity’ (Stock & Watson, 2007). It is very unlikely that the price changes in the data are perfectly multicollinear however it is likely that the predictors would be Imperfect multicollinear. This is due to the commodities being made from similar intermediate goods and therefore have inflationary pressure from the same sources.

Imperfect Multicollinearity does not mean that the OLS is not possible however it will mean that at least one regressors will be imprecisely estimated (Stock & Watson, 2007). To manage the Imperfect Multicollinearity Penalised Regression Methods (PRM) - Ridge and LASSO approach was implemented.

Ridge Regression (RR) is a means of estimating regression coefficients when data are high-dimensional and/or contain correlated variables (Hoerl & Kennard, 1970). Least Absolute Shrinkage and Selection Operator (LASSO) regression, is a shrinkage and variable selection method for regression models, which is well sorted to overcoming overfitting issue, in terms of the number of variables ultimately included in the model. (Ranstam & Cook, 2018) This overestimation of how well the model performs in terms of using the included variables to explain the observed variability is known as optimism bias.

The regression model to be solved was:

Equation 33 Cross Price and Income Regression

The approaches were implemented using GLMNET package in R[[17]](#footnote-17). The difference between the approaches in this package is elastic net mixing parameter, (Hastie, et al., 2023), which has a range where is a ridge model and is a LASSO model. The results can be compared by the figures produced by GLMNET, present in Table 11. This shows that the LASSO has outperformed the Ridge method.

|  |  |  |
| --- | --- | --- |
| Commodity Group | LASSO | Ridge |
| **Commodity 1** | 0.952 | 0.932 |
| Commodity 2 | 0.952 | 0.932 |
| Commodity 3 | 0.952 | 0.932 |
| Commodity 4 | 0.952 | 0.932 |
| Commodity 5 | 0.952 | 0.932 |
| Commodity 6 | 0.952 | 0.932 |
| Commodity 7.1 | 0.952 | 0.932 |
| Commodity 7 - Other | 0.952 | 0.932 |
| Commodity 8 | 0.952 | 0.932 |
| Commodity 9 | 0.952 | 0.932 |
| Commodity 10 | 0.952 | 0.932 |
| Commodity 11 | 0.952 | 0.932 |
| Commodity 12 | 0.952 | 0.932 |

Table 11 LASSO vs RIDGE r2

However further refinement of the model is required as the results are counterintuitive and more likely a modelling anomaly than new insight. The full table of estimators can be found in Annex E: Charts and Results, Table 18 and Table 19.

Model interpretations:

* Majority of commodities have a zero-price elasticity of demand.
* All Commodities are supplementary good for Commodity 1 (Food & non-alcoholic drinks) as the estimator is positive. However, the estimator of Commodity 1 for Commodity 1 is also positive, implying all things being equal a price increase in commodity increases spending in all commodities which is not possible.
* A price change in Commodity 12 has no impact on the quantity of any commodity consumed including that commodity which is unlikely and contradicts the previous regression models.

Although this attempt at calculating the cross-price elasticities via regression modelling has not been successful this work has still been included in the paper. The reason is that it is the believe of the author further work in on this method will yield insights into consumer behaviour that should be considered when constructing the UF for a CGEM.

### Regression Results

The consumer behaviour that regression modelling has highlighted that should be included in a UF include:

* Logarithmic relationship between income and quantity in commodities 1, 2, 4 & 8
* Although the LASSO regression work was inclusive it has yield that not all cross price elasticises will be negative

## Conclusion

This paper was an investigation into the use of CGEM as a policy analysis tool for domestic intervention into an advance manufacturing sector. Due to the weakness discussed above a satisfactory model to appraise the policy decision has not been able to be completed. The first conclusion must be for the ongoing policy decision of intervention level into the advanced manufacturing sectors the previous analytical framework should be continued to be used.

However, this does not mean that this approach could not be utilised with additional work. It would require collaboration with analysis with additional skills in programming, data science and specific trade data knowledge.

The results of this paper conclude that a future model structure should:

* Make use of the multi-year Cobb-Douglas Production Function Parameters,
* Include intermediate good values from ONS,
* Model an exogenous constant saving rate,
* Establish the labour - primary factor costs (salary) based on ONS data,
* Incorporating a tax system, ideally the simplest either a Poll Tax or flat rate of income tax and cooperation tax policy,
  + The justification of a simple tax system is that it increases the likelihood that the model’s equilibrium can be shown to be unique.
* Model government expenditure as an increased consumption of sector goods based on ONS Input-Output table data,
* Include economic data of UK key trade partners at a minimum, recommended would be all trade partners,
* Have a presentation style appropriate for wider audiences to mitigate the risk that the model is perceived as a ‘black box’,
* Include further refinement of the PRM for the cross-price elasticity of demand which should be incorporated to a new UF, alongside the logarithmic income elasticity of demand.

This paper also recommends that further regression analysis is completed on automotive production against vehicle production figures, as discussed in Annex B: Technical: 7 - Wage Premium Uncertainty.

The impact of improving CGEM modelling is not only on the policy decision making but an opportunity to bring DIT and BEIS analytical teams together. This paper has highlighted the challenges with completing a successful CGEM which could be overcome by utilising the skills and knowledge with the DIT team.

## Annex A: Dedication

This paper is dedicated to my wife for her love and support through the master’s program. Without you none of this would have been possible and sorry for the exam the week of our wedding.

## Annex B: Technical

1. Standard Industrial Classification (SIC)

SIC was first introduced into the UK in 1948 for classifying business establishments (ONS, 2009) and can be used across multiple ONS datasets as a method of managing data disaggregated by production sector.

The classifications are in a hierarchy structure, section (letter), division (2-digit), group (3-digit), class (4-digit) and subclass (5-digit). There are 39 classifications used in parts of this report modelling, which are:

|  |  |  |
| --- | --- | --- |
| **Code** | **Description** | **New Code** |
| A01 | 01: Crop and animal production, hunting and related service activities | A |
| A02 | 02: Forestry and logging | A |
| A03 | 03: Fishing and aquaculture | A |
| B | B: MINING AND QUARRYING | A |
| C10T12 | 10-12: Manufacture of food, drink, and tobacco | B |
| C13T15 | 13-15: Manufacture of textiles, wearing apparel, leather, and leather products | C |
| C16 | 16: Manufacture of wood and of products of wood and cork | C |
| C17 | 17: Manufacture of paper and paper products | C |
| C18 | 18: Printing and reproduction of recorded media | C |
| C19 | 19: Manufacture of coke and refined petroleum products | C |
| C20 | 20: Manufacture of chemicals and chemical products | C |
| C21 | 21: Manufacture of basic pharmaceutical products and pharmaceutical preparations | C |
| C22 | 22: Manufacture of rubber and plastic products | C |
| C23 | 23: Manufacture of other non-metallic mineral products | C |
| C24 | 24: Manufacture of basic metals | C |
| C25 | 25: Manufacture of fabricated metal products, except machinery and equipment | C |
| C26 | 26: Manufacture of computer, electronic and optical products | C |
| C27 | 27: Manufacture of electrical equipment | C |
| C28 | 28: Manufacture of machinery and equipment | C |
| C29 | 29: Manufacture of motor vehicles, trailers, and semi-trailers | D |
| C30 | 30: Manufacture of other transport equipment | C |
| C31\_32 | 31-32: Manufacture of furniture and other manufacturing | C |
| C33 | 33: Repair and installation of machinery and equipment | A |
| D | D: ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY | E |
| E | E: Water supply; sewerage, waste management and remediation activities | E |
| F | F: Construction | E |
| G | G: Wholesale and retail trade; repair of motor vehicles and motorcycles | E |
| H | H: Transportation and storage | E |
| I | I: Accommodation and food service activities | E |
| J | J: Information and communication | E |
| K | K: Financial and insurance activities | E |
| L | L: Real estate activities | E |
| M | M: Professional, scientific, and technical activities | E |
| N | N: Administrative and support service activities | E |
| O | O: Public administration and defence; compulsory social security | E |
| P | P: Education | E |
| Q | Q: Human health and social work activities | E |
| R | R: Arts, entertainment, and recreation | E |
| S | S: Other service activities | E |

1. Further Reduction

When completing CGEM the following groupings of industry sectors will be used.

|  |  |
| --- | --- |
| **New Code** | **Definition** |
| **A** | Raw Materials, Manufacturing input |
| **B** | Food goods |
| **C** | Manufactured Non-Food Goods |
| **D** | Automotive Production |
| **E** | Services and Other Goods |

1. Production Function 1

This section gives the mathematical derivation of the linear regression model used to calculate the parameters of the CD production function for each section of the economy. This is an adaption of the methodology for two factor production function found in [**reference**]. This two-factor method has been replicated for SIC 29 to give:

If you assume constant return to scale, then .

Equation 34 Multivariable Linear Regression Model of CD

The constants in this equation can be estimated by multi-variable linear regression which was complete in R. The simple linear regression used to calculate the parameters can return answer in the whole range . Therefore, for some of the sectors there may exist better estimators of the function outside the bounds of the parameters, .

#### Linear Regression Results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** |  | **Alpha** | **Beta** | **Gamma** |
| A01 | 1.364 | 0.414 | 0.111 | 0.475 |
| A02 | 1.602 | 0.188 | 0.353 | 0.458 |
| B | 1.113 | 0.666 | 0.118 | 0.215 |
| C18 | 1.058 | 0.587 | 0.259 | 0.154 |
| C19 | 1.282 | 0.698 | 0.072 | 0.229 |
| C22 | 0.941 | 0.512 | 0.445 | 0.043 |
| C23 | 1.261 | 0.396 | 0.47 | 0.135 |
| C25 | 1.677 | 0.11 | 0.538 | 0.353 |
| C28 | 1.036 | 0.466 | 0.459 | 0.075 |
| C31\_32 | 0.843 | 0.754 | 0.22 | 0.026 |
| D | 0.873 | 0.752 | 0.087 | 0.162 |
| F | 1.758 | 0.571 | 0.099 | 0.331 |
| H | 1.111 | 0.314 | 0.565 | 0.121 |
| N | 0.952 | 0.597 | 0.384 | 0.02 |
| O | 1.094 | 0.138 | 0.63 | 0.232 |
| P | 1.248 | 0.319 | 0.358 | 0.323 |
| Q | 1.904 | 0.293 | 0.244 | 0.462 |

Table 12 Linear Regression Modelling Results

1. Production Function 2

To find the parameters for the sectors that linear regression failed other techniques were required. The aim was to find the local minimum Mean Squared Error (MSE) within the parameter range. Given the level of accuracy the modelling is going to use, the first approach was to use a brute force search method. Calculate the Mean square Error (MSE) of all possible combination of at 2-digits (i.e., 0.23, 0.76, etc).

This approach would give a local minimum within the discrete search space, that would be a false local minimum of the continuous search space. i.e. it is the minimum MSE for the finite discrete set of parameter searched over.

Where the bar denotes the found estimators satisfying the equation:

,

However, this was too computationally intensive and only one result was found after 18 hours of run time. There were 22 sectors that needed finding (c16 day runtime).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** | **A** | **Alpha** | **Beta** | **Gamma** |
| A03 | 0.95 | 0.8 | 0.01 | 0.199 |

1. Production Function 3

The next search method to find the local minimums was an algorithmic search based on shrinking spheres. The starting point for the algorithm is the central point, .

* The step 1 is to create new set of points are selected at random on the surface of the sphere centred at the start point.
* The radius of the sphere decreases with each iteration of the algorithm.
* The MSE of each element of the set is calculated.
* The element with the smallest MSE is the new start point, return to step 1.

The algorithm is illustrated in Figure 4. The code can be found in LM Regression 7.R, the name of the functions in Figure 4 correspond to those in the code.

A diagram of a function

Description automatically generated

Figure 12 Flow Diagram of Search Algorithm

#### Production Function Results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** | **A** | **Alpha** | **Beta** | **Gamma** |
| A01 | 3.91 | 0.41 | 0.11 | 0.48 |
| A02 | 4.96 | 0.19 | 0.35 | 0.46 |
| A03 | 29.92 | 0.41 | 0 | 0.58 |
| B | 3.04 | 0.67 | 0.12 | 0.22 |
| C10T12 | 7.23 | 0.7 | 0.03 | 0.27 |
| C13T15 | 8.06 | 0.75 | 0.18 | 0.06 |
| C16 | 3.3 | 0.88 | 0.07 | 0.05 |
| C17 | 141.98 | 0.29 | 0.57 | 0.13 |
| C18 | 2.88 | 0.59 | 0.26 | 0.15 |
| C19 | 3.6 | 0.7 | 0.07 | 0.23 |
| C20 | 4.82 | 0.81 | 0.13 | 0.06 |
| C21 | 84.22 | 0.41 | 0.49 | 0.09 |
| C22 | 2.56 | 0.51 | 0.44 | 0.04 |
| C23 | 3.53 | 0.4 | 0.47 | 0.13 |
| C24 | 3 | 0.87 | 0.07 | 0.06 |
| C25 | 5.35 | 0.11 | 0.54 | 0.35 |
| C26 | 18.37 | 0.62 | 0.31 | 0.08 |
| C27 | 3.57 | 0.87 | 0.08 | 0.06 |
| C28 | 2.82 | 0.47 | 0.46 | 0.07 |
| C29 | 8.08 | 0.73 | 0.2 | 0.06 |
| C30 | 28.15 | 0.58 | 0.4 | 0.02 |
| C31\_32 | 2.32 | 0.75 | 0.22 | 0.03 |
| C33 | 67.14 | 0.4 | 0.43 | 0.16 |
| D | 2.39 | 0.75 | 0.09 | 0.16 |
| E | 6.48 | 0.8 | 0.14 | 0.06 |
| F | 5.8 | 0.57 | 0.1 | 0.33 |
| G | 37.54 | 0.37 | 0 | 0.63 |
| H | 3.04 | 0.31 | 0.56 | 0.12 |
| I | 5.76 | 0.81 | 0.13 | 0.06 |
| J | 16.51 | 0.66 | 0.28 | 0.06 |
| K | 162.35 | 0.32 | 0.62 | 0.07 |
| L | 289.52 | 0.15 | 0.48 | 0.37 |
| M | 60.66 | 0.43 | 0.48 | 0.09 |
| N | 2.59 | 0.6 | 0.38 | 0.02 |
| O | 2.99 | 0.14 | 0.63 | 0.23 |
| P | 3.48 | 0.32 | 0.36 | 0.32 |
| Q | 6.71 | 0.29 | 0.24 | 0.46 |
| R | 17.81 | 0.61 | 0.31 | 0.07 |
| S | 62.86 | 0.41 | 0.48 | 0.12 |

Table 13 Linear Regression and Search Algorithm Results

These coefficients could be used in any future CGEM modelling without the bias of having a single benchmark year.

1. Household expenditure data

The groupings of commodity and services in the ONS 'Household expenditure by gross income decile group’ are:

|  |  |
| --- | --- |
| Code | Group (Description) |
| 1 | Food & non-alcoholic drinks |
| 2 | Alcoholic drinks, tobacco & narcotics |
| 3 | Clothing & footwear |
| 4 | Housing(net)[[18]](#footnote-20), fuel & power |
| 5 | Household goods & services |
| 6 | Health |
| 7 | Transport |
| 8 | Communication |
| 9 | Recreation & culture |
| 10 | Education |
| 11 | Restaurants & hotels |
| 12 | Miscellaneous goods & services |

Table 14 ONS Household Expenditure Commodity groups

1. Wage Premium Uncertainty

Wage premium is calculated by taking the counterfactual salary from the likely salary post intervention in the automotive sector. The automotive salary cannot be compared directly to the national mean as it does not account for the regional distribution of automotive salaries. The current methodology does not make assumptions about what sectors the new economic activity has displaced. Therefore, the counterfactual salary is the local salary for the area of the new job.

Data on salary for different sectors and location is provided by the Annual Survey of Hours and Earnings[[19]](#footnote-21) from ONS. ASHE data is disaggregated by TTWA or by industry but not by both simultaneously. Therefore, it is not possible to directly know the automotive wage premium in each TTWA from the dataset. Therefore, a counterfactual salary must be constructed by the weight average across different TTWA.

Not all automotive jobs are the same with different skill, experience and NVQ levels required. A higher skill automotive job will by higher but so will the counterfactual salary. The ASHE data is disaggregated by percentiles therefore for the weight average each percentile of automotive data is compared to each percentile of the local salaries.

This is an updated version of the wage premium calculation previous presented in this master (Lambert, et al., 2021).

|  |  |
| --- | --- |
|  | Equation 35 Auto Wage Premium Calculation |

Where:

is the number of employees in TTWA

is the salary of in TTWA for percentile

is the auto salary in percentile

the number of TTWAs

the number of percentiles

This has been calculated for SIC code 29100 which covers production of automotive vehicles, from 2015 -2022. The data is recorded annually in the price year of that data. Therefore, the GDP deflator has been used to make the premiums comparable.

Figure 13 Average wage premium of automotive production

The automotive industry has been hit by external shocks recently these include the Global financial crisis (2009), the trade uncertainty resulting from Brexit (2016) and COVID (2020). These dates have been highlighted in Figure 14 which is IHS data[[20]](#footnote-22).

Figure 14 Total Annual Vehicle Production

This reduction in production since 2016 well maintaining similar capital capacity could have resulted in the depression of the automotive salaries. It would hold therefore that as production recovers as if historically has that the wage premium will follow. To test this hypothesis thoroughly a wage premium vs production regression could be modelled on all the data from 2000 to present.

## Annex C: Code Blocks

A qr code on a white background

Description automatically generatedAll of the R code used for this project can be found at:

<https://github.com/James-Lambertcoding/Dissertation>

Alternative please scan the QR code for access.

This GITHUB also contains all of the data used and charts produced in this paper.

The code split into two sections:

1. R Model – CGEM
2. R Model – Regressions

The code in section 1 covers the CGEM.

|  |  |
| --- | --- |
| **CGEM Model** | **Associated Code** |
| Model 1 | 11\_Model\_1\_Calibration.R |
| Model 2 | 12\_Model\_2\_Calibration.R |
| Non-Unity Wage and 39 Commodity Model | 00\_Main\_Model\_v1.R (which runs)   * 01\_Packages.R * 02\_read\_data\_v2.R * 02a\_Start\_point\_data.R * 03\_Calibration\_v3.R * 04\_CGE\_Functions\_v\_2.R |

Table 15 GCEM Code Mapping

The Non-Unity Wage and 39 Commodity Model was more complex and therefore the code blocks were sub-divided to better handle version control and enable easier Quality Assurance of the code.

The code in section 2 covers regression analysis of UF via investigating Household expenditure and regression of the Production Function.

|  |  |
| --- | --- |
| **Regression Model** | **Associated Code** |
| Price and Income Models for Utility Function | Income\_Price\_Elascity\_v1.R |
| LASSO and Ridge Functions | Income\_Price\_Elascity\_v1.R |
| Production Function Regression | LM Regression 7.R |
| Production Function search Function | LM Regression 7.R |

Price and Income Models for Utility Function Charts are in Output\_2 and split into:

|  |  |
| --- | --- |
| **Folder** | **Associated Code** |
| Plots Linear Income | Commodity Spend vs Income with Linear Regression Line |
| Plots Log Income | Commodity Spend vs Income with Logarithmic Regression Line |
| Plots No Regression Lines | Commodity Spend vs Income with No Regression Line |
| Plots Price | Commodity Spend vs Price with No Regression Line |

## Annex D: Project Management

1. Risk Register

The below risk register is based upon the methodology used in project management within DBT and specific the programme management of the ATF. The risks are measured against likelihood and impact on a 5-level RAG (Red, Amber, Green) scale. These metrics are combined to give a RAG Rating by the risk rating matrix, any rating Amber or above requires a mitigation in place.

A chart with different colors and text

Description automatically generated with medium confidence

Figure 15 Risk Rating Matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Risk** | **Description** | **Impact** | **Likelihood** | **Rating** | **Mitigation** |
| Unavailability of data | Although the data was going to be sourced from ONS open platform there is no guarantee that the disaggregation available would be suitable | Amber/Red | Green/Amber | Amber | There are private data providers that DBT has contracts with which could be used to procure additional data sources |
| Data loss | Due to corruption of hard drives or mishandling of version control there was a loss of work either completely or sections. | Amber/Red | Green/Amber | Amber | The dissertation and code have been backed up across multiple devices and cloud platforms including, personal computer, DBT servers and Google cloud.  Version Control has been implemented using GIT hub to ensure previous states of the code are retrievable. Adds additional redundancy to stop complete data loss.  Comments used exhaustively through out to ensure doing what new parts were doing |
| Policy change | Could there be a change in strategic direction for the policy area meaning that the policy question being answer was no longer relevant. | Amber | Green | Green/ Amber | N/A This work provides additional benefits besides implications for a policy decision |
| Modelling not completable | There is no guarantee that a complete CGEM would be codable within the time of the project | Amber /Red | Amber | Amber/Red | The policy intervention will be assessed by the current methodology within the department. This ensure that the ability to provide robust analysis for policy decision is not solely dependent on this project. |
| Subject matter difficulties | Barriers to completing the work because of difficulties in using the CGEM approach | Red | Amber | Amber/ Red | The author has ensured good working relationship with subject matter experts within the department to provide guidance and |
| Time constraints | Balancing work on project, work and live. Could other commitments stop the completion of the project. | Amber/Red | Amber/Green | Amber | A project timeline was created to ensure that time was divided across all the aspects of the project.  This would ensure that blockers in one aspect of the project do not hold back reaching a suitable conclusion.  Ensure that there is flexibility to include more time/days on to the project |

Table 16 Project Risk Register

1. Project plan

A close-up of a chart

Description automatically generated

Figure 16 Project plan

The project plan was created to ensure that enough time remain to complete tasks and evaluate when contingency days were required to complete steps. One of the main uses was to know when a certain activity had to be stopped (particularly in the modelling) to ensure time to complete the other stages. This enable understanding the trade off between continuing coding/modelling and focusing on completing the write up.

## Annex E: KSBs

Table 13 below is taken from (Institute for Apprenticeships, 2019) as presented by Queen Mary University of London.

|  |  |  |  |
| --- | --- | --- | --- |
| **High Level** | **individual KSBs** | **Pass/Distinction** | **Mapping** |
| **Applied economic analysis** | **S1** Apply micro-economic and macro-economic theories and modelling, including econometric, to inform a range of business and policy decisions. **S2** Convert the policy or other question into a tractable appraisal, evaluation or other analysis drawing on the most appropriate analytical method, including non-market valuation methods. Analysis considers, inter alia: the counterfactual, opportunity cost, risk and uncertainty and how to estimate discount rates and costs of capital. **S3** Critically assess available information sources and judge validity and usefulness for the issue at hand; clean and manipulate data; be aware of data limitations and explain them; clearly describe and present data using data visualisation techniques; and draw out and explain policy and business implications to clients | Apply their economic knowledge to inform a range of business and/or policy decisions. | [Impact/Benefit justification](#_Impact/Benefit_justification) shows that economic knowledge has been applied to a business area and policy decision. |
| Assess and argue what an appropriate method and data sources (s) are and identifies any limitations in the policy/business situation. | The ONS data was identified as the appropriate data source for the project because of the multiple years of data and the reliability. The limitations of the data are discussed in [Data Limitations](#_Data_Limitations) |
| Give a clear explanation of the assumptions made in their analysis and argue effectively for why these are appropriate. | The desired modelling assumption for the CGEM are presented in [Desired Modelling Assumptions](#_Desired_Modelling_Assumptions) with further explanation given to explain reason and justification for them. To ensure they were presented clearly a table format was used in Table 1 |
| Choose a presentation style and data visualisation tools that effectively describe the analysis and draw out recommendations for policy/business decisions. | Multiple plots and maps have been used through the project to illustrate the analysis: particularly:  Figure 2  Figure 4  Figure 5 |
| Identifies data relevant to the issue and the limitations of that data when judging validity and usefulness | The relevant data for the project is the input output tables whose weakness are is discussed in [Data Limitations](#_Data_Limitations). Within this section there is a discussion of the data usefulness and why in the CGEM approach it should continue to be used. In the conclusion the limitations of the data are considered. |
| Evaluate how limitations in the method/data selected could be improved upon and judge what the risks are for the project’s conclusions. | The paper contains a complete section on the modelling weakness, which inform the conclusion of the paper. There is an in-depth mathematical derivation of the weakness to the CD utility function used in the initial modelling, specifically the inability for it to handle income elasticity of demand, in [Utility Function (UF)](#_Utility_Function) |
| Explore a range of ways to improve their assumptions through new analysis and create convincing arguments to support their judgements. | The weakness of the initial utility function analysis and production function assumption are explored, and alternate versions are provided. [Production Function](#_Production_Function) & [Utility Function](#_Utility_Function). |
| Make predictions of the likely impact of their recommendations on the business/policy situation | Provided a prediction on the impact of this project on the analytical working environment between ex-DBT and ex-BEIS analysts, [Conclusion](#_Conclusion) |
| **Project management and planning** | **S5** Scope areas of work identifying objectives, analytical methods, resources required and potential delivery risks.Able to recognise when complementary expertise is required e.g., scientists, other social scientists, and data specialists. | Set out a clear project scope and had the correct resources to deliver the requirements of the project | This evidence by the inclusion of [Project Aims](#_Project_Aims) and [Output Objectives](#_Output_Objectives). The resources were in place to work on the project as best could give time constraints by reaching out to Trade Analysis with Ex-DIT |
| Understood the risks to the project and set out evidence of how these were mitigated during the project. | Evidence by the Risk Register:  Table 6 Project Risk Register |
| Have, where required, drawn on other sources of expertise and opinion to inform their results and ensure maximum impact. | The initial stages of developing this work required collaborating with trade Analyst in ex-DIT who have the sources and the expertise to inform the work in the project. |
| Demonstrate how the learning they have generated during the project could be used to inform future projects and/or the wider workplace. | This is evidenced in the recommendations in the [Conclusion](#_Conclusion) |
| Devised processes for interdisciplinary working or tools to improve the effectiveness of interdisciplinary working. | The join up and buy in from trade analysts has improved interdisciplinary working and enabled a smoother transition to a joint DBT analytical working environment. |
| **Effective communication** | **S7** Clearly communicate economic principles and concepts to non-economists; present trade-offs and uncertainties and articulate these clearly; frame advice, drawing on knowledge of stakeholders' positions, for maximum impact | Communicate complex economic ideas to a non-economist audience. | This is evidenced by the [Non-Technical Summary](#_Non-Technical_Summary) |
| Explore options and trade-offs. | Exploration of options and trade-offs is given in [Structural Choices](#_Structural_Choices) and by the paper covering multiple models and different function that could be used in the approach. |
| Set out key uncertainties. | The two key uncertainties in the modelling approach are set out in Uncertainties. |
| Frames advice showing awareness of how stakeholders will react to analysis or recommendations. | The advice in the conclusion is continue working with the ex-DIT analysts shows how to best land the advice in the project by anticipating stakeholder reactions. |
| Demonstrates that they can tailor their communication approach to the needs of different audiences. | This is evidenced by the [Non-Technical Summary](#_Non-Technical_Summary) and the language in the [Conclusion](#_Conclusion) being tailored to the non-analytical audience. There is the inclusion of maps and charts to help communicate to those who benefit from visual explanations. Evidenced by  Figure 2  Figure 4  Figure 5 |
| **Horizon scanning** | **S6** Use horizon scanning methodologies to anticipate new trends, opportunities and challenges that may influence outcomes of interest to client. | Show how trends, future opportunities and future challenges will affect their analysis. | This evidence by the inclusion of the description of Governmental Departmental Changes which has helped motivate this project. |
| Demonstrate how their conclusions are resilient to future events or represent least regret solutions. | In [Data Limitations – SIC Codes](#_Data_Limitations_–) there is a discussion of how future changes to the automotive industry will impact the modelling. It presents a method to build resilience into the modelling for these future events. |
| **Maintaining quality standards** | **S8** Design Quality Assurance processes and implement these, following organisational best practices, and drawing on sources of external expertise; critically assess economic analysis and improve it. **B1** Ethical conduct: analyst attributes sources and ideas to their originator; provides honest advice on all relevant aspects to an issue; avoids bias. **B4** Rigour: demonstrates a commitment to detail. | Devise a robust quality assurance process for their work to ensure that analysis and written outputs are accurate and error free. | When creating the code of the models in the project’s steps have been taken to allow robust quality assurance. This includes:  Proper version control by using GITHUB.  A thorough comments to enable others to follow.  For complex modelling the code blocks have been broken out, evidence by |
| Clearly set out and reference all the sources they use, including both data sources and the source of ideas. | Referencing system has been used and [Bibliography](#_Bibliography) provided to set out clearly the sources of ideas. The sources of data are provided with links to original data sets where possible. Evidence by footnotes 1, 6, 8, and 12. There has been necessary redaction of sensitive sources. |
| Selected the appropriate level of detail necessary to achieve the required output. | The paper is presented at different levels, [Non-Technical Summary](#_Non-Technical_Summary) and [Conclusion](#_Conclusion) for policy focussed readers as well as a detailed derivation for analytical interested readers, [Model Derivation](#_Model_Derivation). This will enable the recommendation of the paper to be understood by all readers. |
| • Compare their results with results from other methods or studies to check validity of results and conclusions. | The results from the equilibrium model were checked against the delta function to test their validity. The project is looking at an alternative methodology for a policy analysis, which has been used to check the conclusion against. |
| • Describe the steps taken to ensure that their analysis is free from bias. |  |
| • Ensure that inconvenient facts or analyses that do not fit their argument are addressed. | This paper included a through exploration of the weakness of the approach and provides full details of the limitation in the author ability to complete a full complex CGEM. Therefore, show that all facts and analysis has been included |
| • Draw on new sources of expertise that are external to their organisation which substantially improve the robustness of, or insight from, findings. | The |
| • Challenge conventional wisdom and/or existing approaches in a sensitive and effective way. | This paper is its challenge the conventional wisdom of the approach to sector intervention analysis by incorporating a new approach. However, it balances the |

Table 17 QMUL KSBs

## Annex F: Charts and Results

Income and spend regression chart results:

A screenshot of a graph

Description automatically generated

A screenshot of a graph

Description automatically generated

A screenshot of a graph

Description automatically generated

A graph with blue and red dots

Description automatically generated

1. **LASSO Modelling Results**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **LASSO** | **Regressors** | | | | | | | | | | | | | |
| **Commodity Groups** | Income | Price 1 | Price 2 | Price 3 | Price 4 | Price 5 | Price 6 | Price 7.1 | Price 7 Other | Price 8 | Price 9 | Price 10 | Price 11 | Price 12 |
| Commodity 1 | 82.709 | 0.067 | -0.16 | 0 | 0 | 0 | 0 | 0 | -0.063 | 0 | -0.149 | -0.09 | -0.016 | 0 |
| Commodity 2 | 16.41 | 0.012 | 0 | 0 | 0 | -0.059 | 0 | 0 | 0 | 0 | -0.039 | 0 | -0.039 | 0 |
| Commodity 3 | 53.105 | 0.065 | -0.013 | 0 | -0.275 | -0.046 | 0 | 0 | 0 | 0 | 0 | -0.241 | 0 | 0 |
| Commodity 4 | 81.737 | 0.049 | 0.164 | 0 | -0.266 | 0 | 0 | -0.107 | 0 | 0 | 0 | -0.224 | -0.065 | 0 |
| Commodity 5 | 34.398 | 0.074 | 0 | 0 | 0.125 | 0 | -0.401 | 0 | 0 | -0.105 | 0 | 0 | 0.112 | 0 |
| Commodity 6 | 7.745 | 0.012 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.075 | 0 | 0 |
| Commodity 7.1 | 21.002 | 0.072 | 0.006 | 0 | 1.235 | 0 | -0.433 | 0 | -1.107 | 0 | 0 | 0.63 | 0.019 | 0 |
| Commodity 7 - Other | 43.179 | 0.11 | 0 | -0.062 | 0 | 0 | 0 | -0.107 | 0 | -0.19 | -0.018 | -0.061 | 0 | 0 |
| Commodity 8 | -0.857 | 0.019 | -0.066 | 0 | 0 | 0 | 0 | 0 | 0.047 | 0 | 0 | 0.065 | 0 | 0 |
| Commodity 9 | 65.39 | 0.17 | -0.037 | 0 | 0.211 | 0 | 0 | 0 | -0.121 | 0 | 0 | -0.82 | 0 | 0 |
| Commodity 10 | -173.829 | 0.029 | 0.26 | 0.012 | 0.028 | 0.59 | -1.402 | 0 | 1.399 | -1.058 | 1.746 | 0.596 | -0.46 | 0 |
| Commodity 11 | 31.517 | 0.109 | -0.238 | -0.137 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.062 | 0 | 0 |
| Commodity 12 | 20.819 | 0.091 | -0.074 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.075 | -0.044 | -0.04 | 0 |

Table 18 LASSO Model Regressor Results

The cells highlighted in blue represent the model’s estimation of linear quantity responsiveness to self-price changes.

1. **Ridge Regression Modelling Results**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **LASSO** | **Regressors** | | | | | | | | | | | | | |
| **Commodity Groups** | Income | Price 1 | Price 2 | Price 3 | Price 4 | Price 5 | Price 6 | Price 7.1 | Price 7 Other | Price 8 | Price 9 | Price 10 | Price 11 | Price 12 |
| Commodity 1 | 63.852 | 0.061 | 0 | -0.009 | 0 | 0 | -0.191 | 0 | 0 | 0 | -0.016 | 0 | 0 | 0 |
| Commodity 2 | 15.075 | 0.011 | 0 | 0 | 0 | -0.053 | 0 | 0 | 0 | 0 | -0.03 | 0 | -0.037 | 0 |
| Commodity 3 | 1.748 | 0.058 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.008 | 0 | -0.024 |
| Commodity 4 | 62.764 | 0.045 | 0 | 0 | -0.171 | 0 | 0 | 0 | 0 | 0 | 0 | -0.063 | -0.055 | 0 |
| Commodity 5 | 0.12 | 0.067 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Commodity 6 | 2.182 | 0.011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.015 | 0 | 0 |
| Commodity 7.1 | -6.815 | 0.065 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.003 | 0 |
| Commodity 7 - Other | 9.553 | 0.099 | 0 | 0 | 0 | 0 | -0.091 | 0 | 0 | 0 | 0 | 0 | 0 | -0.047 |
| Commodity 8 | 4.675 | 0.017 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Commodity 9 | -4.299 | 0.152 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Commodity 10 | -5.749 | 0.026 | 0 | 0 | 0 | -0.045 | 0 | 0 | 0.054 | 0 | 0 | 0 | -0.004 | 0 |
| Commodity 11 | 10.027 | 0.098 | -0.034 | 0 | 0 | -0.137 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Commodity 12 | 3.935 | 0.082 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.045 | 0 | 0 | 0 |

Table 19 Ridge Regression Model Regressor Results

The cells highlighted in blue represent the model’s estimation of linear quantity responsiveness to self-price changes.

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1. [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\_data/file/  
   1063330/Green\_Book\_2022.pdf](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1063330/Green_Book_2022.pdf) [↑](#footnote-ref-1)
2. Gigafactory is the term for a site making cells to go into the batteries of EVs. [↑](#footnote-ref-2)
3. Redacted internal HMG source. [↑](#footnote-ref-3)
4. Redacted internal HMG source. [↑](#footnote-ref-4)
5. https://www.gov.uk/government/publications/levelling-up-the-united-kingdom [↑](#footnote-ref-5)
6. Redacted internal HMG source and reasoning. [↑](#footnote-ref-6)
7. <https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/datasets/ukinputoutputanalyticaltablesdetailed> [↑](#footnote-ref-7)
8. This code runs: 01\_Packages.R & 02\_read\_data\_v2.R & 02a\_Start\_point\_data.R & 03\_Calibration\_v3.R & 04\_CGE\_Functions\_v\_2.R. [↑](#footnote-ref-8)
9. [https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/expenditure/datasets/  
   householdexpenditurebygrossincomedecilegroupuktablea4](https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/expenditure/datasets/householdexpenditurebygrossincomedecilegroupuktablea4) [↑](#footnote-ref-9)
10. Excluding mortgage interest payments, council tax and Northern Ireland rates [↑](#footnote-ref-10)
11. Although the data in the Education group had gaps unlike others, so likely not significant as a result as other groups. [↑](#footnote-ref-11)
12. survey of 4,000 consumers by leading supermarket media and promotions business Shopmium. [↑](#footnote-ref-12)
13. Source: <https://my.ihs.com/> [↑](#footnote-ref-13)
14. <https://www.gov.uk/government/collections/gdp-deflators-at-market-prices-and-money-gdp> [↑](#footnote-ref-14)
15. <https://www.ons.gov.uk/economy/inflationandpriceindices/datasets/consumerpriceindices> [↑](#footnote-ref-15)
16. Excluding mortgage interest payments, council tax and Northern Ireland rates [↑](#footnote-ref-16)
17. <https://cran.r-project.org/web/packages/glmnet/index.html> [↑](#footnote-ref-17)
18. Excluding mortgage interest payments, council tax and Northern Ireland rates [↑](#footnote-ref-20)
19. <https://www.ons.gov.uk/surveys/informationforbusinesses/businesssurveys/annualsurveyofhoursandearningsashe> [↑](#footnote-ref-21)
20. This data is from a license provided to DBT and therefor the actual figures have been altered randomly but the inference taken from the chart remains true. [↑](#footnote-ref-22)